DEPARTMENT OF CIVIL ENGINEERING, UNIVERSITY OF
ENGINEERING AND TECHNOLOGY, LAHORE

DESIGN OF BEAM FOR TORSION

Engr. Ayaz Waseem Malik
Lecturer/ Lab Engineer

DESIGN OF BEAM FOR TORSION

1. IMPORTANT TERMINOLIGIES

A_{cp}	Area enclosed by outside perimeter of concrete cross-section
A_g	Gross area of concrete section. For a hollow section, A_g is the area of the concrete only and does not include the area of void(s)
A_l	Total area of longitudinal reinforcement to resist torsion
$A_{l(min)}$	Minimum area of longitudinal reinforcement to resist torsion
A_o	Gross area enclosed by the shear flow path
A_{oh}	Area enclosed by centerline of the outermost closed transverse torsional reinforcement
A_t	Area of one leg of closed stirrup resisting torsion
A_v	Area of steel for resisting shear
b_t	Width of that part of cross-section containing the closed stirrups resisting torsion
f_c'	Specified compressive strength of concrete
f_y	Specified yield strength of reinforcement
f_{yt}	Specified yield strength of transverse reinforcement
N_u	Factored axial force normal to cross-section occurring simultaneously with V_u or T_u
P_{cp}	Outside perimeter of concrete cross-section
P_h	Perimeter of centerline of the outermost closed transverse torsional reinforcement
t	Wall thickness of hollow section
T_{cr}	Cracking torque
T_n	Nominal torsional moment strength
T_u	Factored torsional moment at section
V_c	Nominal shear strength provided by concrete
V_n	Nominal shear stress
V_u	Factored shear force
λ	Modification factor reflecting the reduced mechanical properties of lightweight concrete, all relative to normalweight concrete of the same compressive strength.

2. TORSION

Torsion occurs appreciably in many structures, such as in the main girders of bridges, which are twisted by transverse beams or slabs. It occurs in buildings where the edge of a floor slab and its main beams are supported by a spandrel beam running between the exterior columns. This situation is shown in the Figure 1.1, where the floor beam acts as off-center load and tends to twist the spandrel beam laterally. Buildings in which the center of mass and center of rigidity do not coincide are called asymmetrical structures. In such structures earthquakes can cause dangerous torsional forces.

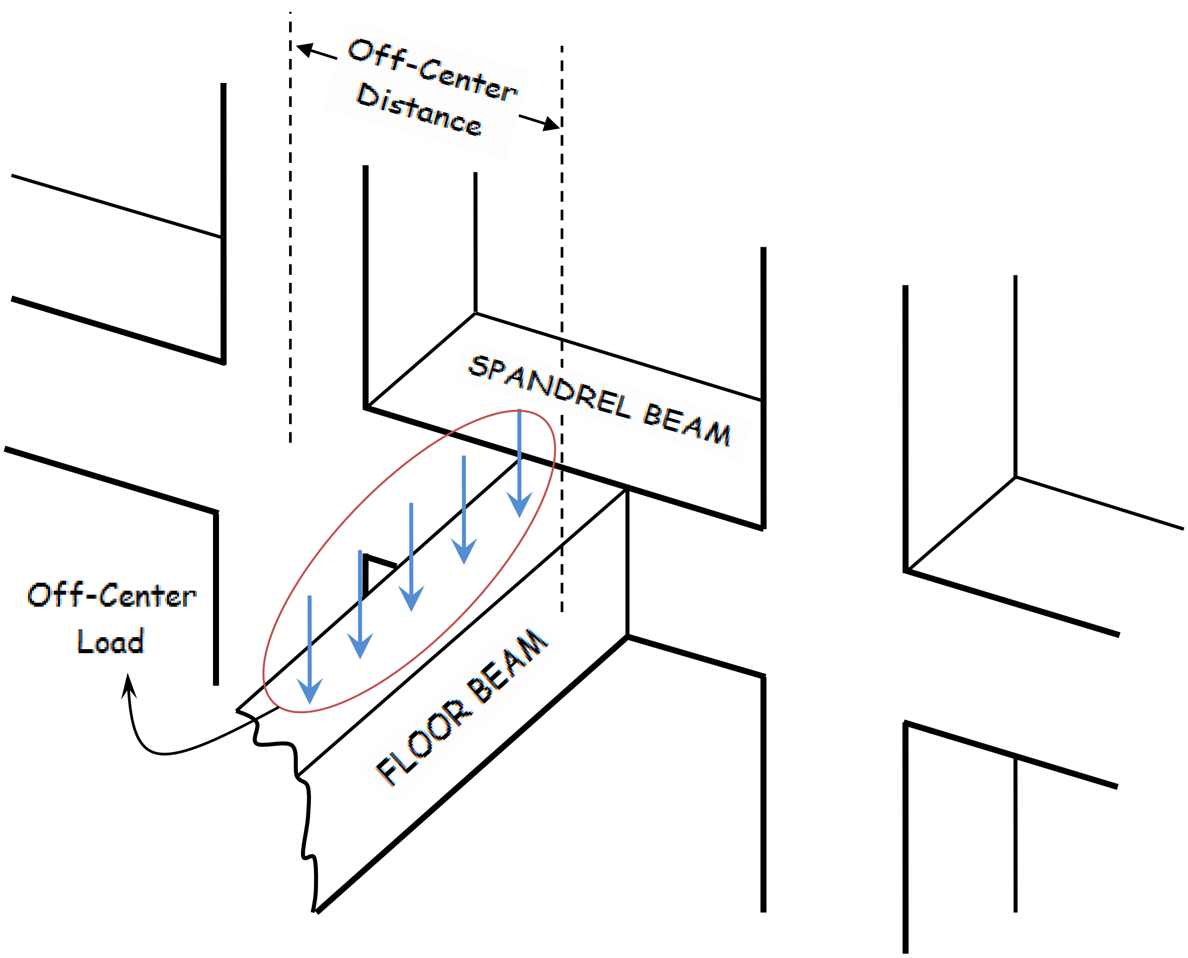


Figure 1.1 **Off-center load causing torsion in the spandrel beam**

If a plain concrete member is subjected to pure torsion, it will crack and fail along 45^o spiral lines due to diagonal tension corresponding to the torsional stresses. Although the diagonal tension stresses produced by twisting are very similar to those caused by shear, they will occur on all faces of a member. Maximum shears and torsional forces may occur in areas where bending moments are small. For such cases, the interaction of shear and torsion can be particularly important as it relates to design.

3. THRESHOLD TORSION

According to ACI 11.5.1, it shall be permitted to neglect torsion effects if,

$$T_u < \phi\, 0.083\, \lambda \sqrt{f_c'} \left(\frac{{A_{cp}}^2}{P_{cp}}\right) \tag{1.1}$$

For members subjected to an axial tensile or compressive force it shall be permitted to neglect torsion effects if,

$$T_u < \phi\, 0.083\, \lambda \sqrt{f_c'} \left(\frac{{A_{cp}}^2}{P_{cp}}\right) \sqrt{1 + \frac{N_u}{0.33 A_g\, \lambda \sqrt{f_c'}}} \tag{1.2}$$

Cracking Torque

The cracking torsion under pure torsion T_{cr} is derived by replacing the actual section with an equivalent thin-walled tube with a wall thickness t prior to cracking of 0.75 A_{cp} /p_{cp} and an area enclosed by the wall centerline A_o equal to 2A_{cp} /3. Cracking is assumed to occur when the principal tensile stress reaches 0.33λ. In a nonprestressed beam loaded with torsion alone, the principal tensile stress is equal to the torsional shear stress, $\tau = T/(2A_{ot})$. Thus, cracking occurs when τ reaches 0.33λ, giving the cracking torque T_{cr} (According to R11.5.1) as

$$T_{cr} = 0.33\, \lambda \sqrt{f_c'} \left(\frac{{A_{cp}}^2}{P_{cp}}\right) \tag{1.3}$$

Using Eq. 1.3 in Eq. 1.1 we get,

$$T_u < \phi\, \frac{1}{4}\, T_{cr} \tag{1.4}$$

The above equation states that *Torsional effects can be neglected if the factored torsional moment does not exceed approximately one-quarter of the cracking torque* T_{cr} .

4. MOMENT REDISTRIBUTION

Equilibrium Torsion

For a statically determinate structure, there is only one path along which a torsional moment can be transmitted to the supports. This type of torsional moment, which is referred to as *equilibrium torsion*, cannot be reduced by redistribution of internal forces or by a rotation of member. Equilibrium torsion is illustrated in Figure 1.2.

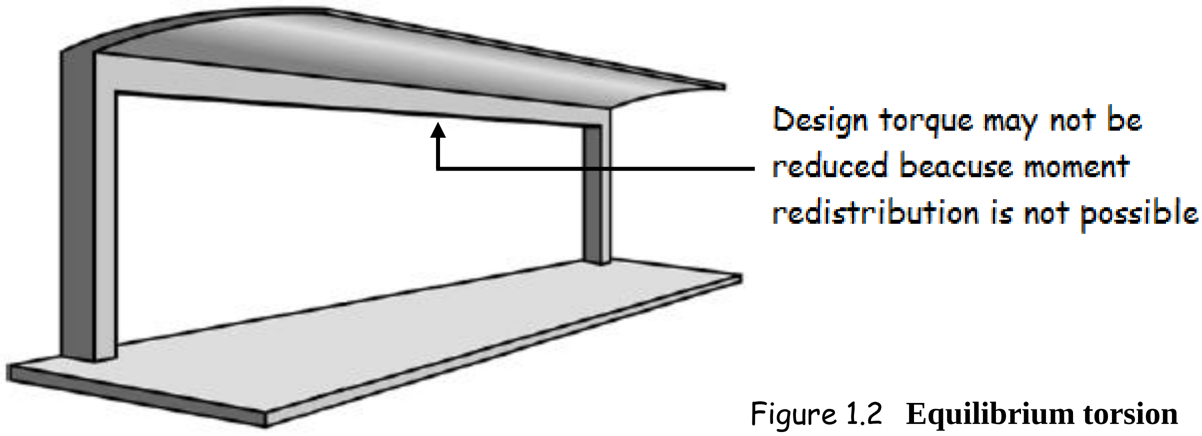


Figure 1.2 **Equilibrium torsion**

Compatibility Torsion

The torsional moment in a particular part of a statically indeterminate structure may be substantially reduced if that part of the structure cracks under torsion and/or rotates. The result will be a redistribution of forces in the structure. This type of torsion, which is illustrated in Figure 1.3, is referred to as *compatibility torsion*, in the sense that part of structure in question twists in order to keep the deformations of the structure compatible.

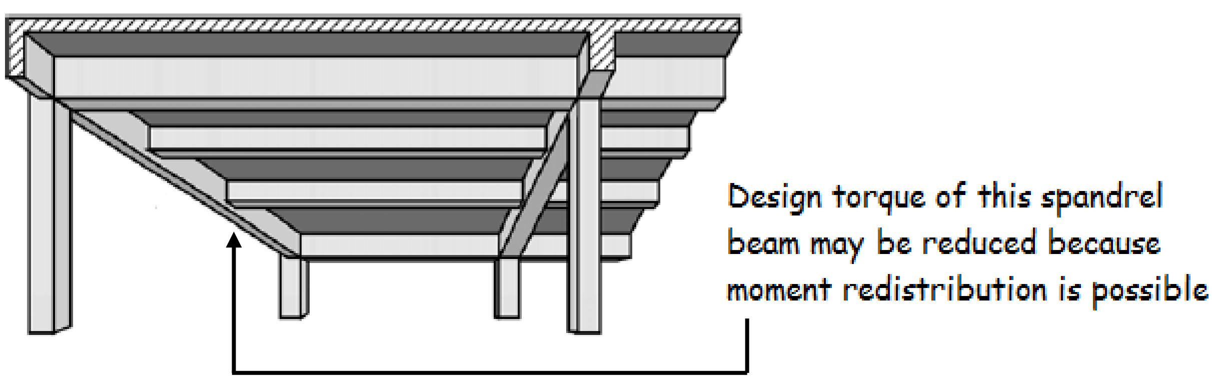


Figure 1.3 **Compatibility Torsion**

In a statically indeterminate structure where reduction of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, the maximum factored torsional moment T_u shall be permitted to be reduced to the following values (According to ACI 11.5.2.2),

$$T_u = \phi\, 0.33\, \lambda \sqrt{f_c{}'} \left(\frac{{A_{cp}}^2}{P_{cp}}\right) \tag{1.5}$$

For members subjected to an axial tensile or compressive force it shall be permitted to neglect torsion effects if,

$$T_u < \phi\, 0.33\, \lambda \sqrt{f_c{}'} \left(\frac{{A_{cp}}^2}{P_{cp}}\right) \sqrt{1 + \frac{N_u}{0.33 A_g\, \lambda \sqrt{f_c{}'}}} \tag{1.6}$$

When appreciable torsion is present, it may be more economical to select a larger beam than would normally be selected so that torsional reinforcement does not have to be used. Such a beam may very well be more economical than a smaller one with the closed stirrups and additional longitudinal steel required for torsion design.

5. TORSIONAL STRESSES

The torsional stress adds to the shear stress on one side of a member and subtract from them on the other. This situation is illustrated in the Figure 1.4.

Torsional stresses are quite low near the center of a solid beam. Because of this, hollow beams are assumed to have almost exactly the same torsional strengths as solid beams with the same outside dimensions.

In solid sections the shear stresses due to torsion Tu are concentrated in an outside tube of the member as shown in Figure 1.4(b), while the shear stresses due to V_u are spread across the width of the solid section.

After cracking, the resistance of concrete to torsion is assumed to be negligible. The tension cracks tend to spiral around members located at approximately 45^o angles with the longitudinal edges of those members. Torsion is assumed to be resisted by an imaginary space truss located in the outer tube of concrete of the member. Such a truss is shown in Figure. 1.5. The longitudinal steel in the corners of the member and the closed transverse stirrups act as tension members in the truss, while the diagonal concrete between the stirrups acts as struts of compression members. The cracked concrete is still capable of taking compression stresses.

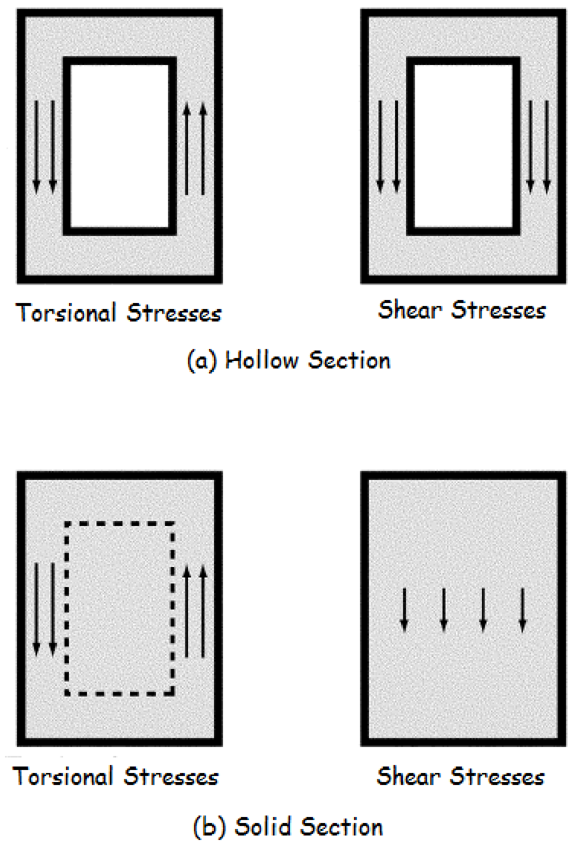


Figure 1.4 **Addition of torsional and shear stresses**

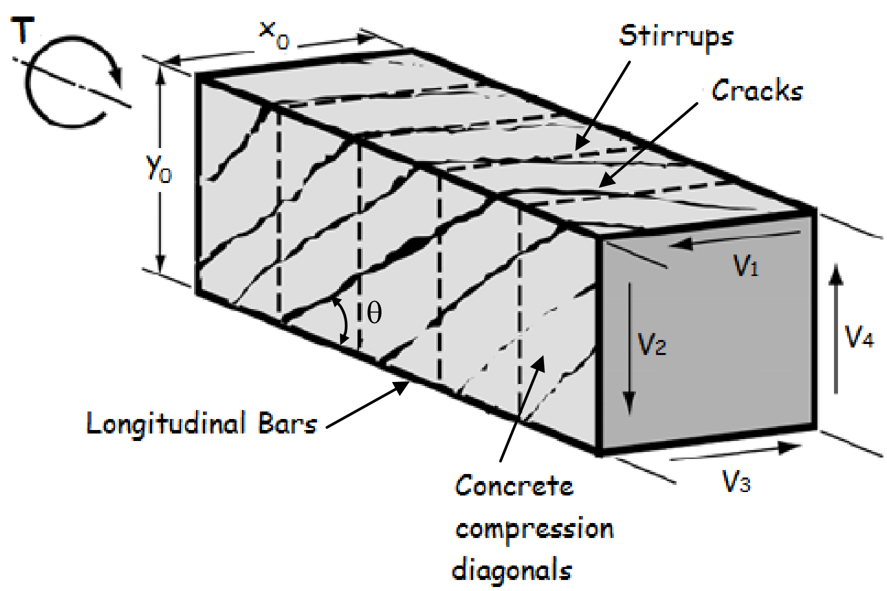


Figure 1.5 **Space truss analogy**

6. TORSIONAL MOMENT STRENGTH

According to ACI 11.5.3.1, the cross-sectional dimensions shall be such that:

(a) For solid section,

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right) \quad (1.7)$$

(b) For hollow section,

$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u P_h}{1.7A_{oh}^2}\right) \leq \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right) \quad (1.8)$$

The size of a cross section is limited for two reasons; first, to reduce unsightly cracking, and second, to prevent crushing of the surface concrete due to inclined compressive stresses due to shear and torsion. In Eq. (1-7) and (1-8), the two terms on the left-hand side are the shear stresses due to shear and torsion. The sum of these stresses may not exceed the stress causing shear cracking plus $0.66\sqrt{f_c{}'}$. If the wall thickness of a hollow section is less than A_{oh}/P_h, the second term in Eq. 1.8 is to be taken not as $T_u P_h/1.7A_{oh}{}^2$ but as $T_u/1.7A_{oh}$, where t is the thickness of the wall where stresses are being checked.

7. TORSIONAL REINFORCEMENT REQUIRED

The torsional strength of reinforced concrete beams can be greatly increased by adding torsional reinforcement consisting of closed stirrups and longitudinal bars. If the factored torsional moment for a particular member is larger than the value given in Eq. 1.1, then the ACI 11.5.5.2 provides an expression to compute the absolute minimum area of transverse closed stirrups that may be used.

$$A_v + 2A_t = 0.062\sqrt{f_c'}\frac{b_w s}{f_{yt}} \geq \frac{0.35 b_w s}{f_{yt}} \quad (1.9)$$

From the knowledge of shear design of beam it is clear that the area A_v obtained is for both legs of a two-legged stirrup. The value A_t, which represents the area of the stirrups needed for torsion, is for only one leg of the stirrup. Therefore the value $A_v + 2A_t$ is the total area of both legs of the stirrups needed for shear plus torsion. According to ACI 11.5.3.6, area of stirrups A_t used for resisting torsion is computed as follows,

$$T_n = \frac{2A_o A_t f_{yt}}{s} cot\theta \quad (1.10)$$

where A_o shall be determined by analysis except that it shall be permitted to take A_o equal to $0.85A_oh$; θ shall not be taken smaller than 30° nor larger than 60°. It shall be permitted to take θ equal to 45° for nonprestressed members.

Where torsional reinforcement is required, the total area of longitudinal reinforcement to resist torsion, A_l, shall be computed according to ACI 11.5.3.7

$$A_l = \left(\frac{A_t}{s}\right) P_h \left(\frac{f_{yt}}{f_y}\right) cot^2\,\theta \tag{1.11}$$

The minimum total area of longitudinal torsional reinforcement, $A_{l(min)}$, shall be computed according to ACI 11.5.5.3 as,

$$A_{l(min)} = \frac{0.42\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) P_h \frac{f_{yt}}{f_y} \tag{1.12}$$

where A_t/s shall not be less than $0.175b_w/f_{yt}$.

Maximum torsion generally acts at the ends of beams, and a result the longitudinal torsion bars should be anchored for their yield strength at the face of the supports. To do this it may be necessary to use hooks.

8. OTHER ACI CODE PROVISIONS

1) ACI 11.5.6.1

Maximum spacing, S_{max}, for transverse torsion reinforcement is smaller of;

a) 300 mm.
b) $P_h/8$.

2) ACI 11.5.6.2

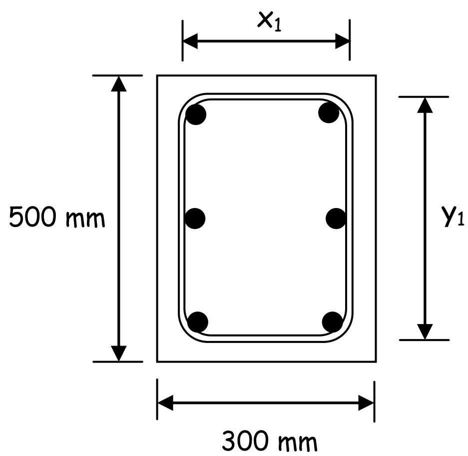
The longitudinal reinforcement required for torsion must be distributed around the inside perimeter of the closed stirrups with a maximum spacing of 300 mm. There shall be at least one longitudinal bar or tendon in each corner of the stirrups. Longitudinal bars shall have a diameter at least 0.042 times the stirrup spacing, but not less than No. 10.

3) ACI 11.5.6.3

Torsional reinforcement must be provided for a distance not less than $b_t + d$ beyond the point where it is theoretically no longer required. The term b_t represent the width of that part of the member cross-section which contains the torsional stirrups.

Design Example

Design the torsional reinforcement for the rectangular section shown in Figure 1.6, for which f_c' = 30 MPa and f_y = 420 MPa, V_u = 160 kN, T_u = 10 kN-m and A_s required for M_u is 1100 mm^2.



Solution

Step 1: Check for torsional reinforcement requirement

$A_{cp} = (500)(300) = 150000\ mm^2$

$P_{cp} = (2)(500 + 300) = 1600\ mm^2$

$$\phi\ 0.083\ \lambda \sqrt{f_c'} \left(\frac{{A_{cp}}^2}{P_{cp}}\right) = (0.75)(0.083)(1)\sqrt{30}\left(\frac{(150000)^2}{1600}\right)$$

$= 4794712\ N\text{-}mm = 4.8\ kN\text{-}m\ <\ T_u = 10\ kN\text{-}m$

∴ Torsional reinforcement is required

Step 2: Calculation of sectional properties

$Diameter\ of\ stirrups = 13\ mm$

$Clear\ cover = 40\ mm$

$$x_1 = 300 - 2(40) - 2\left(\frac{13}{2}\right) = 207\ mm$$

$$y_1 = 500 - 2(40) - 2\left(\frac{13}{2}\right) = 407\ mm$$

$A_{oh} = (207)(407) = 84249\ mm^2$

$A_o = 0.85A_{oh} = (0.85)(84249) = 71612\ mm^2$

Assuming bottom reinforcement consists of #25 bars,

$d = 500 - 40 - 13 - \left(\frac{25}{2}\right) = 434.5\ mm$

$P_h = 2(x_1 + y_1) = 2(207 + 407) = 1228\ mm$

Step 3: Check for adequacy of concrete cross-section

According to ACI 11.2.1.1,

$V_c = 0.17\lambda\sqrt{f_c'}b_w d$ …………………………………………………………….(ACI 11.3)

$V_c = 0.17(1)\sqrt{30}(300)(434.5) = 121372.6\ N = 121.4\ kN$

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7A_{oh}^2}\right)^2} = \sqrt{\left(\frac{160 \times 10^3}{(300)(434.5)}\right)^2 + \left(\frac{(10 \times 10^6)(1228)}{1.7(84249)^2}\right)^2} = 1.59\ N/mm^2$$

$$\phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right) = 0.75\left(\frac{121.4 \times 10^3}{(300)(434.5)} + (0.66)\sqrt{30}\right) = 3.41\ \text{N}/mm^2$$

$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$ ……………………………………..(ACI 11.18)

$1.59\ N/mm^2 < 3.41\ N/mm^2$

∴ Section is sufficient to support T_u

Step 4: Transverse torsional reinforcement

$T_n = \frac{T_u}{\phi} = \frac{10}{0.75} = 13.33\ kN.m$

$T_n = \frac{2A_o A_t f_{yt}}{s} cot\theta$ ……………………………………………………………..(ACI 11.21)

$\frac{A_t}{s} = \frac{13.33 \times 10^6}{2(71612)(420)\cot(45^o)} = 0.222\ mm^2/mm\ \ for\ 1\ leg\ of\ stirrup$

Step 5: Shear reinforcement

$$\frac{1}{2}V_c = \frac{1}{2}(121.4) = 60.7\ kN \quad < \quad V_u = 160\ kN$$

∴ Shear reinforcement is required

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{160 - (0.75)(121.4)}{0.75} = 91.93\ kN$$

$$\frac{A_v}{s} = \frac{V_s}{f_y d} = \frac{91.93 \times 10^3}{(420)(434.5)} = 0.504\ mm^2/mm\ \ for\ two\ legs\ of\ stirrups$$

Step 6: Design of Stirrups

$$\frac{2A_t}{s} + \frac{A_v}{s} = 2(0.222) + 0.504 = 0.948\ mm^2/mm\ \ for\ two\ legs\ of\ stirrups$$

$Using\ \#13\ stirrups, \quad A_s = 133\ mm^2$

$$s = \frac{(2)(133)}{0.948} = 281\ mm$$

$s_{max} = lesser\ of;\quad$ 1. $300\ mm$

2. $P_h/8 = 1228/8 = 154\ mm$ ⇐ Governs

Taking, $s = 150\ mm$

∴ Stirrups provided are,

#13 @ 150 mm c/c

Step 7: Design of Longitudinal Reinforcement

$$A_l = \left(\frac{A_t}{s}\right) P_h \left(\frac{f_{yt}}{f_y}\right) cot^2\,\theta = (0.222)(1228)\left(\frac{420}{420}\right)(1)^2 = 272\ mm^2$$

$$\frac{0.175 b_w}{f_{yt}} = \frac{0.175(300)}{420} = 0.125\ mm^2/mm \quad < \quad \frac{A_t}{s} = 0.222\ \ mm^2/mm$$

$$A_{l(min)} = \frac{0.42\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) P_h \frac{f_{yt}}{f_y}$$

$$A_{l(min)} = \frac{(0.42)\sqrt{30}(150000)}{420} - (0.222)(1229)\left(\frac{420}{420}\right) = 549\ mm^2 \ > \ A_l$$

$$Taking, \qquad A_l = 549\ mm^2$$

Additional longitudinal steel is provided in four inside corners of stirrups and vertically in between. So Total longitudinal steel provided in beam is,

$$Top\ steel = \frac{549}{3} = 183\ mm^2$$

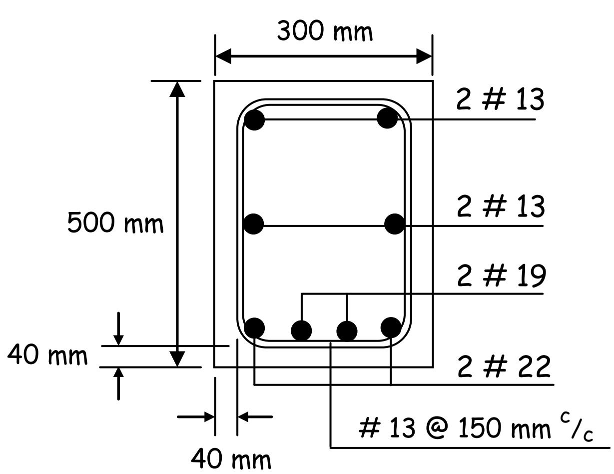
$$Mid\ steel = \frac{549}{3} = 183\ mm^2$$

$$Bottom\ steel = \frac{549}{3} + 1100 = 1283\ mm^2$$

∴ Longitudinal steel provided is,

Top Steel	Mid Steel	Bottom Steel
2 # 13 Bars	2 # 13 Bars	2 # 22 + 2 # 19 Bars

Step 7: Detailing



REFERENCES

1. Building Code Requirements for Structural Concrete (ACI 318M-08) and Commentary
2. Design of Reinforced concrete, 7th Edition, by Jack C. McCormac and James. K. Nelson
3. Reinforced concrete design, 4th Edition, by W. H. Mosley and J. H. Bungey